

Progression Towards a Written Method for Subtraction

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of addition
- not commutative i.e. $5 - 3$ is not the same as $3 - 5$
- not associative i.e. $10 - 3 - 2$ is not the same as $10 - (3 - 2)$

YR

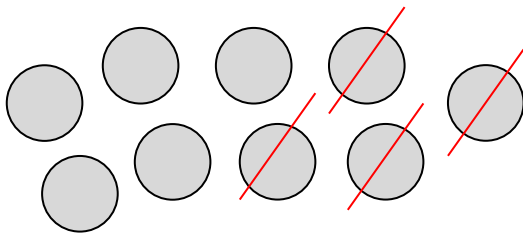
Early Learning Goal:

Children count reliability with number from 1-20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they subtract two single digit numbers and count all/on to find the answer.

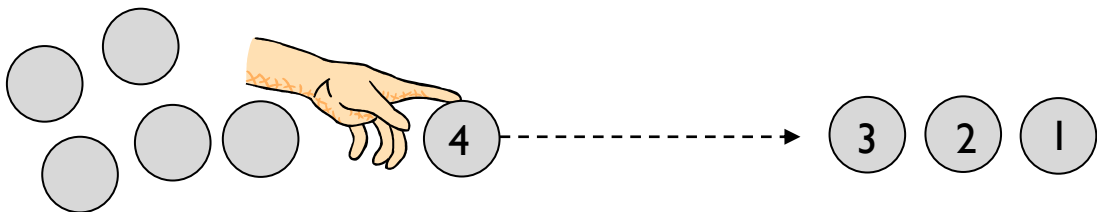
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Taking away

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g. $9 - 4$.



For illustration purposes, the amount being taken away are show crossed out. Children would be encouraged to physically remove these using touch counting.



By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

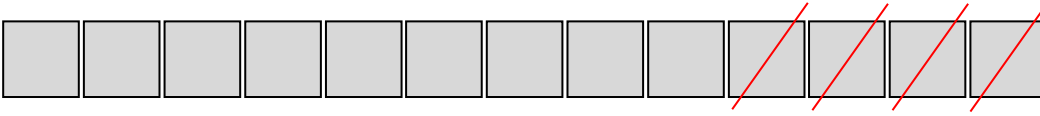
Those who are ready may record their own calculations.

Y1

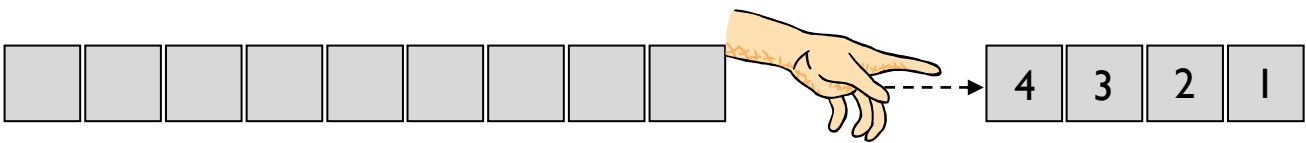
End of Year Objective:

Subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

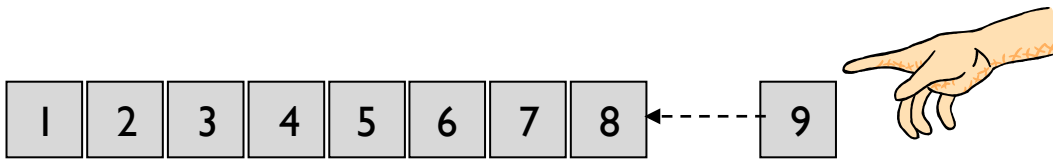
Children will continue to use practical equipment and taking away strategies. To avoid the need to exchange for subtraction at this stage, it is advisable to continue to use equipment such as counters, cubes and the units from the Base 10 equipment, but not the tens, e.g. $13 - 4$



Touch count and remove the number to be taken away, in this case 4.



Touch count to find the number that remains. Additionally number lines are used to count back. Then when the children are confident, the children will begin to use mental strategies.



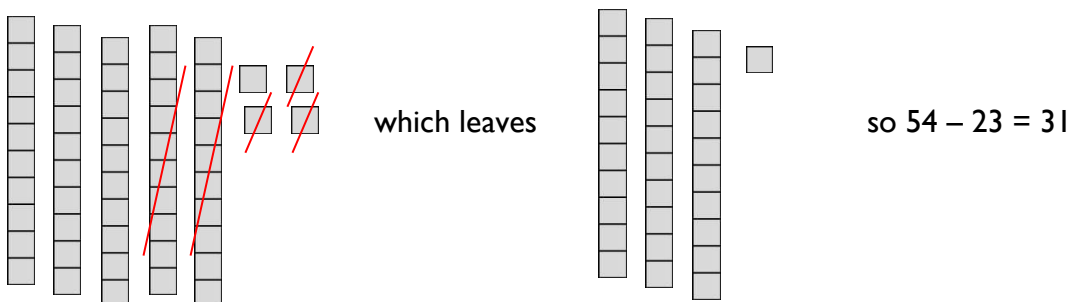
Number lines and base 10 are also used to further develop their understanding of subtraction.

Y2

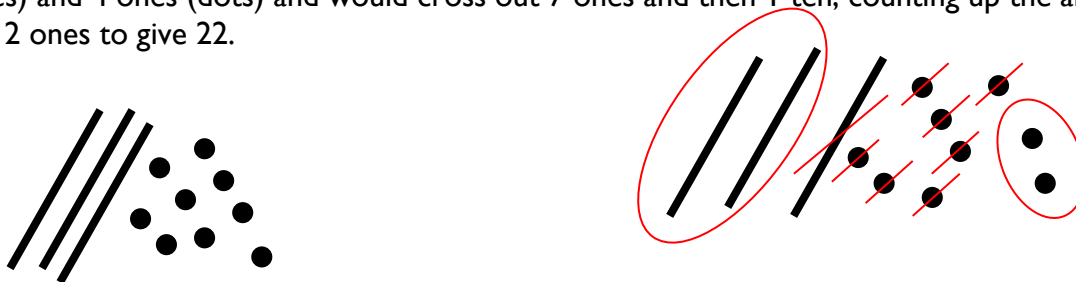
End of Year Objective:

Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers.

Children will continue to use the Base 10 equipment to support their calculations, still using a take away, or removal, method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount. For example, to calculate $54 - 23$, children would count out 54 using the Base 10 equipment (5 tens and 4 ones). They need to consider whether there are enough ones to remove 3, in this case there are, so they would remove 3 ones and then two tens, counting up the answer of 3 tens and 1 one to give 31.



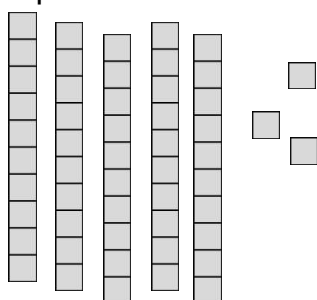
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the ones), e.g. to calculate $39 - 17$ children would draw 39 as 3 tens (lines) and 4 ones (dots) and would cross out 7 ones and then 1 ten, counting up the answer of 2 tens and 2 ones to give 22.



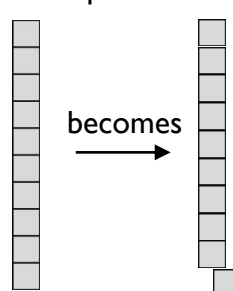
Circling the tens and ones that remain will help children to identify how many remain.

When the amount of ones to be subtracted is greater than the ones in the original number, an exchange method is required. This relies on children's understanding of ten ones being an equivalent amount to one ten. To calculate $53 - 26$, by using practical equipment, they would count out 53 using the tens and ones, as in Step 1. They need to consider whether there are enough ones to remove 6. In this case there are not so they need to exchange a ten into ten ones to make sure that there are enough, as in step 2.

Step 1

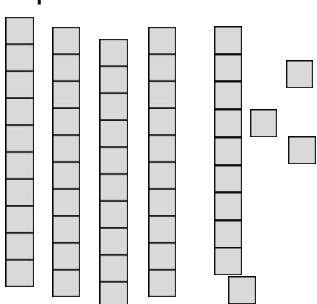


Step 2

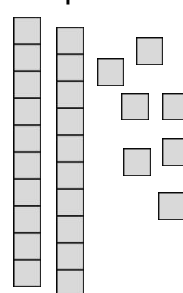


The children can now see the 53 represented as 40 and 13, still the same total, but partitioned in a different way, as in step 3 and can go on to take away the 26 from the calculation to leave 27 remaining, as in Step 4.

Step 3

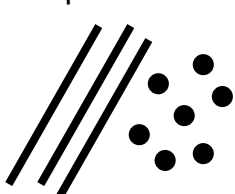


Step 4

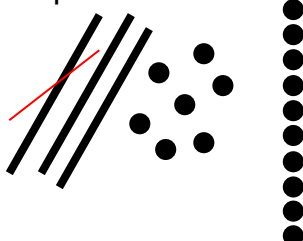


When recording their own drawings, when calculating $37 - 19$, children would cross out a ten and exchange for ten ones. The exchanged ten will be crossed out so children recognise this has not been subtracted. Drawing the ones in a vertical line, as in Step 2, ensures that children create ten ones and do not get them confused with the ones that were already in place. Additional number lines both blank and numbered, 100 square, column subtraction to further embed before mental strategies.

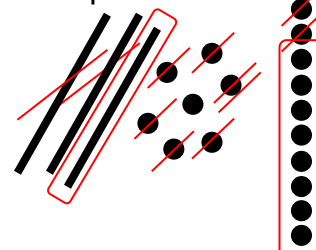
Step 1



Step 2



Step 3



Circling the tens and ones that remain will help children to identify how many remain.

Y3

End of Year Objective:
Subtract numbers with up to three digits, using formal written method of columnar subtraction.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Children will build on their knowledge of using Base 10 equipment from year 2 and continue to use the idea of exchange. This process should be demonstrated using numerals to show the partitioning and Base 10 materials to represent the first number, removing the ones and tens as appropriate (as with the more informal method in year 2).

Step 1

<hr/>	

Step 2

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Step 3

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Emphasise that the second (bottom) number is being subtracted from the first (top) number rather than the lesser number from the greater.

This will be recorded by the children as:

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array}$$

Children can also use jottings of the Base 10 materials (as in year 2) to support with their calculation, as in the example below.

From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough ones to remove 6. In this case there are not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same ($71 = 70 + 1 = 60 + 11$).

By the end of year 3, children should also extend this method for three digit numbers.

Y4

End of Year Objective:

Subtract numbers with up to 4 digits *and* decimals with one decimal place using the formal written method of columnar subtraction where appropriate.

Children will move to year 4 using whichever method they were using as they transitioned from year 3.

When children are ready, it will lead on to the compact method of decomposition:

$$\begin{array}{r}
 \begin{array}{cccc}
 & 6 & 14 & \\
 & \cancel{7} & \cancel{5} & 14 \\
 4 & & & \\
 - & 3 & 2 & 8 & 6 \\
 \hline
 1 & 4 & 6 & 8
 \end{array}
 \end{array}$$

By the end of year 4, children should be using the written method confidently and with understanding. They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

Y5

End of Year Objective:

Subtract whole numbers with more than 4 digits *and* decimals with two decimal places, including formal written methods (columnar subtraction).

Children should continue to use the decomposition method to solve calculations such as:

$$\begin{array}{r}
 \begin{array}{cccc}
 & 6 & 10 & 12 \\
 & \cancel{7} & \cancel{7} & \\
 - & 3 & 2 & 2 & 6 \\
 \hline
 3 & 8 & 4 & 6
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc}
 & 2 & 13 \\
 & \cancel{3} & \cancel{4} & 12 \\
 - & 1 & . & 7 & 6 \\
 \hline
 1 & . & 6 & 6
 \end{array}
 \end{array}$$

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

Y6

End of Year Objective:
Subtract whole numbers and decimals using formal written methods (columnar subtraction).

Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits.

$$\begin{array}{r}
 \overset{5}{\cancel{6}} \overset{13}{\cancel{4}} | 3 \ 2 \\
 - \quad 4 \ 6 \ 8 \ 1 \\
 \hline
 1 \ 7 \ 5 \ 1
 \end{array}$$

$$\begin{array}{r}
 \overset{3}{\cancel{4}} | \overset{6}{\cancel{7}} . \overset{11}{\cancel{2}} | 0 \\
 - \quad 3 \ 4 . 7 \ 1 \\
 \hline
 3 \ 8 \ 2 . 4 \ 9
 \end{array}$$

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

Below are examples stated in the National Curriculum

Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division

This appendix sets out some examples of formal written methods for all four operations to illustrate the range of methods that could be taught. It is not intended to be an exhaustive list, nor is it intended to show progression in formal written methods. For example, the exact position of intermediate calculations (superscript and subscript digits) will vary depending on the method and format used.

For multiplication, some pupils may include an addition symbol when adding partial products. For division, some pupils may include a subtraction symbol when subtracting multiples of the divisor.

Addition and subtraction

789 + 642 becomes

$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ \small{1 \quad 1} \end{array}$$

Answer: 1431

874 – 523 becomes

$$\begin{array}{r} 874 \\ - 523 \\ \hline 351 \end{array}$$

Answer: 351

932 – 457 becomes

$$\begin{array}{r} \\ 932 \\ - 457 \\ \hline 475 \end{array}$$

Answer: 475

932 – 457 becomes

$$\begin{array}{r} \\ 932 \\ - 457 \\ \hline 475 \end{array}$$

Answer: 475